Eliminate the parameter for the parametric equations $\begin{array}{l} x = 2\cos t \\ y = \cos 2t \end{array}$.

SCORE: _____ / 15 PTS

3 $\cos t = \frac{1}{2}x$

6 $y = 2\cos^2 t - 1$

4 $y = 2\left(\frac{1}{2}x\right)^2 - 1$

2 $y = \frac{1}{2}x^2 - 1$

BJ and CJ were working on their polar graphing partner quiz. **SCORE: _____ / 40 PTS**

On the question about the polar equation $r = 2\sqrt{3} - 4\sin 3\theta$, they determined correctly that the symmetry tests $(-r, \theta)$, $(r, -\theta)$, $(-r, \pi - \theta)$ and $(-r, -\theta)$ do **NOT** indicate that the graph is symmetric.

POLE AXIS AXIS $\theta = \frac{\pi}{2}$

[a] **Using their results, along with the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLE: $(r, \pi + \theta)$

$r = 2\sqrt{3} - 4\sin 3(\pi + \theta)$ 4

$r = 2\sqrt{3} - 4\sin(3\pi + 3\theta)$

$r = 2\sqrt{3} - 4(\underbrace{\sin 3\pi}_{0} \cos 3\theta + \underbrace{\cos 3\pi}_{-1} \sin 3\theta)$

$r = 2\sqrt{3} + 4\sin 3\theta$ 4

Type of symmetry	Conclusion
Over the pole	CAN'T TELL
Over the polar axis	CAN'T TELL
Over $\theta = \frac{\pi}{2}$	SYM

4

$\theta = \frac{\pi}{2}$: $(r, \pi - \theta)$

$r = 2\sqrt{3} - 4\sin 3(\pi - \theta)$ 4

$r = 2\sqrt{3} - 4(\underbrace{\sin 3\pi}_{0} \cos 3\theta - \underbrace{\cos 3\pi}_{-1} \sin 3\theta)$

$r = 2\sqrt{3} - 4\sin 3\theta$ 4

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ 5

[c] Find all angles **algebraically** in the minimum interval in part [b] at which the graph goes through the pole.

5 $2\sqrt{3} - 4\sin 3\theta = 0$

$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$2\frac{1}{2}$ $\sin 3\theta = \frac{\sqrt{3}}{2}$

$3\theta \in \left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$

5 $3\theta = -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

$2\frac{1}{2}$ $\theta = -\frac{4\pi}{9}, \frac{\pi}{9}, \frac{2\pi}{9}$

Find the logarithmic formula for $\sinh^{-1} x$ by solving $x = \sinh y$ for y using the exponential definition and an algebraic substitution $z = e^y$ (or a similar substitution).

SCORE: _____ / 25 PTS

5 $\boxed{x = \frac{e^y - e^{-y}}{2}} = \boxed{\frac{z - \frac{1}{z}}{2}} \cdot \frac{z}{z} = \frac{z^2 - 1}{2z}$ 3

$2xz = z^2 - 1$

5 $\boxed{0 = z^2 - 2xz - 1}$

6 $\boxed{z = \frac{2x \pm \sqrt{4x^2 + 4}}{2}} = \frac{2x \pm 2\sqrt{x^2+1}}{2} = \boxed{x \pm \sqrt{x^2+1}}$ 3

$e^y = x + \sqrt{x^2+1}$

$\boxed{y = \ln(x + \sqrt{x^2+1})} = \sinh^{-1} x$

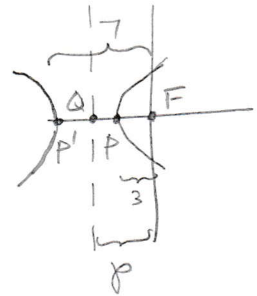
3

A hyperbola has a focus at the pole and vertices with **rectangular** co-ordinates $(-3, 0)$ and $(-7, 0)$. **SCORE: _____ / 25 PTS**

[a] Find polar co-ordinates for the vertices, using positive values of r and θ. **NOTE: You do NOT need to show work.**

2 $\boxed{(3, \pi)}$, $\boxed{(7, \pi)}$ 2

[b] Find the **polar** equation of the hyperbola.



$e = \dfrac{PF}{PQ} = \dfrac{P'F}{P'Q}$

$e = \boxed{\dfrac{3}{p-3} = \dfrac{7}{7-p}}$ 5

$21 - 3p = 7p - 21$

$42 = 10p$

$\boxed{p = \dfrac{21}{5}}$ 3

$\boxed{e = \dfrac{3}{\frac{21}{5} - 3} \cdot \dfrac{5}{5} = \dfrac{15}{21-15} = \dfrac{15}{6} = \dfrac{5}{2}}$ 2

6 $\boxed{r = \dfrac{ep}{1 - e\cos\theta}}$

$r = \boxed{\dfrac{\frac{\cancel{5}}{2} \cdot \frac{21}{\cancel{5}}}{1 - \frac{5}{2}\cos\theta}} \cdot \dfrac{2}{2}$ 3

2 $\boxed{r = \dfrac{21}{2 - 5\cos\theta}}$

AJ throws a football from an initial height of 4 feet, at 18 feet per second, at an angle of 60° with the horizontal. **SCORE: _____ / 15 PTS**
Write parametric equations for the position of the football.

$x = (v_0 \cos\theta)t$

$y = h + (v_0 \sin\theta)t - 16t^2$

$v_0 = 18$

$\theta = 60^\circ$

$h = 4$

5 $x = (18 \cos 60^\circ)t$ $\rightarrow$ $x = 9t$ $2\frac{1}{2}$

5 $y = 4 + (18 \sin 60^\circ)t - 16t^2$ $\rightarrow$ $y = 4 + 9\sqrt{3}t - 16t^2$ $2\frac{1}{2}$

Rewrite $\operatorname{csch}(-\frac{1}{2}\ln x)$ in terms of exponential functions and simplify. **SCORE: _____ / 10 PTS**

$$\boxed{\frac{2}{e^{-\frac{1}{2}\ln x} - e^{\frac{1}{2}\ln x}}}$$ 3

$$= \boxed{\frac{2}{\frac{1}{\sqrt{x}} - \sqrt{x}}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$ 4

$$= \boxed{\frac{2\sqrt{x}}{1-x}}$$ 3

Name the shape of the graphs of the following polar equations. Be as specific as possible. If the graph is a rose curve, state the number of petals.

SCORE: _____ / 20 PTS

[a] $r = 5 + 9\sin\theta$ 4 LIMACON WITH LOOP

[b] $r = 5\sin\theta$ 3 CIRCLE

[c] $r = 5\cos 4\theta$ 4 ROSE CURVE (8 PETALS)

[d] $r = \dfrac{9}{5 + 4\sin\theta}$ 3 ELLIPSE

[e] $r = \dfrac{9}{4 - 5\cos\theta}$ 3 HYPERBOLA

[f] $r = 9 - 4\cos\theta$ 3 CONVEX LIMACON